We give the results of an experimental and theoretical investigation of the thermal resistance of contact between wavy surfaces in a vacuum. It is found that for those values of the geometric parameters of wavy surfaces that are important in practice the thermal contact resistance (TCR) results essentially from contraction of the lines of thermal flux to the areas of macrocontacts.

The majority of studies on TCR [1-3] are based on a consideration of contacts for strictly controlled conditions, when the surfaces form an ideally selected pair. However, the contacting surfaces of the elements of real equipment often deviate from plane surfaces and are wavy [4]. The lack of calculation schemes for joints of this kind causes considerable difficulty in the design of equipment subjected to thermal stress and operating under high-vacuum conditions.

Consider a thermal model based on the effect of dual contraction of heat flux [5]. According to this model, when there is no heat-conducting medium in the contact zone and no oxide film on the contact surfaces, the $T C R$ can be represented in the form of two resistances connected in series: a resistance due to contraction of the heat flux to macroscopic (contour) areas of contact, Rctrma, and a resistance due to contraction of the heat flux to microcontacts, $R_{c t r}$ i, i.e.,

$$
\begin{equation*}
R_{\mathrm{M} \mathrm{~m}}^{\prime}=R_{\mathrm{ctr}_{\mathrm{ma}}}^{\prime}+R_{\mathrm{ctr}_{\mathrm{mi}}}^{\prime} \tag{1}
\end{equation*}
$$

The components of Eq. (1) for the individual microcontacts and macrocontacts when $T_{C}=$ const on the contact area have the form

$$
\begin{equation*}
R_{c t \mathrm{~m}_{\mathrm{mi}}}^{\prime}=\frac{\varphi_{\mathrm{mi}}}{2 a_{\mathrm{mi}}^{\bar{\lambda}_{\mathrm{M}}}}, \quad R_{\mathrm{ctr}}^{\mathrm{ma}}=\frac{\varphi_{\mathrm{ma}}}{2 a_{\mathrm{Ma}}^{\overline{\mathrm{h}}_{\mathrm{M}}}} \tag{2}
\end{equation*}
$$

where the contraction coefficient is approximated by the relations

$$
\begin{equation*}
\varphi_{\mathrm{mi}}=1-1.7 \eta_{3}^{1 / 2}+0.7 \eta_{3}, \varphi_{\mathrm{ma}}=1-1.7 \eta_{2}^{1 / 2}+0.7 \eta_{2} \tag{3}
\end{equation*}
$$

We shall transform Eq. (1), taking account of (2), as applied to the problem of the contact of wavy surfaces, and for this purpose we introduce the following assumptions:

1) all microcontacts of radius $a_{m i}$ lie within contour areas of contact which have radius $a_{m a}$ (Fig. 1);
2) a channel having the shape of a circular cylinder and a radius equal to $\mathrm{L} / 2$, half the wave interval, is arranged coaxially with each macrocontact;
3) the condition $Z \gg L / 2$ is satisfied;
4) all the external surfaces of the cylinders, with the exception of the zone of direct contact and the cylinder endfaces opposite the contact, are adiabatic. We obtain

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Fig. 1. Scheme for contact between wavy surfaces with contact element (macrocontact).



Fig. 2. Comparison of theoretical data (solid curve) and experimental data for a contact pair made of 2 Kh 13 (a) and D16 (b) with wavy surfaces (the conditions under which the experiments were conducted are shown in Table 2).

TABLE 1. Calculated Values of $\mathrm{Retr}_{\mathrm{ma}} / \mathrm{Retr}_{\mathrm{mi}}$

| Metal of contact pair | Method of treatment of the surfaces |  |  |  | a 0 0 0 0 0 0 | \% | 淢 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| D16 | Grinding | 4,5; 3,4 | 20,7 | 106 | 5 | 373 | 158 |
| The same | Same | Same | Same | Same | 50 | Same | 167 |
| 2 Kh 13 | Planing | 12,3; 11,7 | 7,7 | 55 | 5 | 393 | 587 |
| Same | Same | Same | Same | Same | 20 | Same | 611 |
| Same | Same, followed by grinding with paste | 3,2; 2,8 | 7,5 | 56,6 | 5 | 》 | 1010 |

The displacement coefficients $\Omega$ and $\omega$ appearing in (4) physically express the increase in the TCR when the areas of macrocontact and microcontact, respectively, are displaced with respect to the axis of the thermal channels.

TABLE 2. Conditions under which the Experiment was Conducted

|  | Type of treatment | $\begin{gathered} E \\ m^{n} \\ 0 \\ \vdots \\ \stackrel{1}{Ð} \\ B \end{gathered}$ |  |  | $\begin{aligned} & \mathrm{T}_{\mathrm{C}}, \\ & { }^{\circ} \mathrm{K} \end{aligned}$ | Conditions of contact |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | a) Contact pair made of $2 \mathrm{Khl3}$ |  |  |  |  |
| 1 | Grinding | 2.8 | 340 | 4.1; 3.8 | 393 | Loading |
| 2 | Same | same | same | same | same | Unloading |
| 3 | Planing | 0.85 | 55 | 12.3; 11.7 | same | Loading |
| 4 | Same | same | same | same | same | After rolling $\mathrm{P} / \mathrm{E}>$ $5 \cdot 10^{-5}$ |
| 5 | Same | 2.4 | 77 | 10.8;11.8 | 373 | Loading |
| 6 | Grinding | 2.16 | 44.6 | 6.1; 5.7 | same | same |
|  |  | b) Contact pair made of D16 |  |  |  |  |
| 1 | Planing | 2.5 | $25.5$ | 8.1; 8.7 | $373$ | Loading |
| 2 | Planing and finishing on polishing disk | same | same | 2.7; 3.1 | same | Loading |
| 3 | Milling | 1.8 | 53 | 12.1; 11.4 | 403 | same |
| 4 | Same | same | same | same | same | Unloading |
| 5 | Grinding | 2.1 | 69 | 4.1; 3.7 | 373 | Loading |
| 6 | Same | same | same | same | same | Unloading, oxidized surface |
| 7 | Grinding | 2.2 | 106 | 4.5; 3.4 | same | Loading, one surface plane |

The temperature field was investigated on the EDGA installation, using electric-ally conductive paper, for various values of $a / r$ and $d / r$; we obtained an equation describing the effect of displacement of the contact area with respect to the axis of the thermal channel:

$$
\begin{equation*}
\frac{\bar{\lambda}_{\mathrm{MI}}}{1 / R_{\mathrm{M}} r_{\mathrm{mi}}}=\frac{\pi}{2} \frac{r_{0}}{a_{\mathrm{mi}}} \varnothing_{\mathrm{mi}}+3.5\left(\frac{d}{r_{0}}\right)^{2 . \varepsilon} \tag{5}
\end{equation*}
$$

As the results of the investigation indicate, in the working range of values of $\alpha / r$ and $d / r$ the displacement coefficient varies from 1 to 1.4 .

Let us analyze the contribution of the components in Eq. (4) to the total thermal resistance Retrma. We consider the resistance of contraction to macrocontacts, and for this purpose we transform the first term of Eq. (4) to dimensionless form:
where

$$
\frac{1 / R_{\mathrm{ctr}_{\mathrm{ma}}}^{L / 2}}{\bar{\lambda}_{\mathrm{MI}}}=\Phi\left(\eta_{2}\right),
$$

$$
\Phi\left(\eta_{2}\right)=\frac{2}{\pi} \frac{\eta_{2}^{1 / 2}}{\left(1-1.7 \eta_{2}^{1 / 2}+0.7 \eta_{2}\right)}
$$

or, introducing the equivalent thickness $\bar{\Delta}_{\mathrm{ma}}=\bar{\lambda}_{M} /\left(1 / R_{c t_{m a}}\right)$, we obtain

$$
\begin{equation*}
\frac{\bar{\Delta}_{\mathrm{ma}}}{L / 2}=\frac{1}{\Phi\left(\eta_{2}\right)} \tag{6}
\end{equation*}
$$



Fig. 3. Results obtained by processing the experimental data on contact heat exchange in a vacuum for contact pairs made of 2 Kh 13 and D16 with wavy surfaces (the experimental points are indicated in the same way as in Fig. 2). The curve was constructed by the method of least squares.

As the tests on the ÉGDA showed, formula (6) remains correct for the range $n_{2}<$ 0.56 .

From the expression (6) it can be seen that the variation of the TCR as a function of the surface geometry, the physicomechanical properties of the materials, and the load can be synthesized by the variable $\eta_{2}$. To calculate the relative area of contact of wavy surfaces, we make the following approximations: 1) the waves can be simulated by a collection of spherical segments of equivalent radius $r_{m a}$ with uniform distribution along the height [6]; 2) the surface of the waves is covered with spherical microprojections; 3) the waves are elastically deformed; 4) the presence of microroughness does not affect the deformation of the waves; 5) the equation $\left[\mathrm{H}_{\text {mava }} /(\mathrm{L} / 2)\right] \approx[(\mathrm{L} / 2) /$ $2 r_{m_{i}}$ ] ( $i=1$ or 2 ). Then Hertz's formulas [7] for $1-\mu^{2} \approx 0.9$ are transformed to

$$
\begin{equation*}
\pi_{2} \text { masph-pl }=1.65\left[\left(\frac{P}{E}\right)\left(\frac{L / 2}{H \text { maav }}\right)\right]^{2 / 3} \tag{7}
\end{equation*}
$$

for contact between a plane and a wavy surface and

$$
\begin{equation*}
\eta_{2 \mathrm{masph}-\mathrm{sph}}=2.88\left[\left(\frac{P}{E}\right)\left(\frac{L / 2}{\mathrm{H}_{\mathrm{maav}_{1}}+\mathrm{H}_{\mathrm{maav}_{2}}}\right)\right]^{4 / 5} \tag{8}
\end{equation*}
$$

for contact between two wavy surfaces.
In order to determine the contribution made by macrocontact and microcontact resistances, we investigated on a rod-type installation [3] a set of contact pairs made of 2 Kh 13 steel and D16 duralumin under a high vacuum ( $10^{-5}$ torr). The investigations covered specimens with a wide selection of wavy surfaces. From the surface of the specimens, we have profilograms on the basis of which we took a sampling of profiles with regular waviness. In a number of cases the specimens were first subjected to rolling.

The results of theoretical calculation according to formula (6), shown in Fig. 2, and the experimental data indicate that they are in satisfactory agreement in a number of cases. This applies primarily to surfaces with small microroughness, when the conditions of contact are closest to the limiting conditions of the theoretical model, according to which the TCR results essentially from contraction of the curves of thermal flux to macrocontacts. The same is indicated by the reduction in the dispersion of ex-
perimental points in relation to the theoretical curve for surfaces which were rolled prior to the experiment and for surfaces which were very cleanly finished.

As the roughness of the wavy surfaces increases, there is an increase in the contribution made by the resistance of contraction to microcontacts, as a result of which the total contact resistance increases and the experimental points lie above the theoretical curve. This is particularly notable for contact between surfaces with waviness lower than class 3 and roughness lower than class 6 finish. 0xidizing of the surfaces is also accompanied by considerable increase in TCR.

A clear representation of the contribution of the resistances of macroscopic and microscopic areas of contact can be obtained from the formula

$$
\begin{equation*}
\frac{R_{\mathrm{ctr}_{\mathrm{ma}}}^{R_{\mathrm{M}} \mathrm{mi}}}{=} \frac{2}{\pi} \frac{L / 2 \eta_{\mathrm{I}} \Omega}{\Phi\left(\eta_{2}\right) \varphi\left(\eta_{1}\right) a_{\mathrm{mi}}^{\omega}} . \tag{9}
\end{equation*}
$$

Table 1 gives the results of calculations made with expression (9) for specimens made of stainless steel and duralumin, with surfaces of various geometries and over a broad range of loadings. It can be seen that the resistance of the macrocontacts exceeds the resistance of the microcontacts by one order of magnitude or more. If the roughness is increased or the loading is decreased, there is an increase in the contribution made by the resistance of the microcontacts.

Thus, it can be seen that macroconcentrations of heat flux are significant for the TCR of wavy surfaces in a vacuum, and they are decisive for surfaces with waviness higher than class 3 and roughness higher than class 6 finish. For this range of geometric parameters of wavy surfaces, we can trace the relation between the thermal resistances of contraction to macrocontacts and the relative contour area of contact (Fig. 3), which is well approximated by the equation

$$
\begin{equation*}
\frac{\bar{\Delta}_{\mathrm{ma}}}{\bar{L} / \overline{2}}=0.662 \eta_{2}^{-0.65} . \tag{10}
\end{equation*}
$$

Taking account of Eqs. (6)-(8), we can transform (10) to a form more suitable for calculations:

$$
\begin{equation*}
\operatorname{Rctr}_{\text {ma }}=\frac{L / 2 \Omega}{2.1 \bar{\lambda}_{\mathrm{M}}\left(\frac{P}{E}\right)^{0.43}\left(\frac{L / 2}{\mathrm{H}_{\mathrm{mav}}}\right)^{0.43}} \tag{11}
\end{equation*}
$$

for contacts with plane and wavy surfaces, and

$$
\begin{equation*}
\operatorname{Rctr}_{\mathrm{ma}}=\frac{L / 2 \Omega}{3 \bar{\lambda}_{\mathrm{M}}\left(\frac{P}{E}\right)^{0.52}\left(\frac{L / 2}{\mathrm{H}_{\mathrm{ma} \mathrm{av} 1}+\mathrm{H}_{\mathrm{ma} \text { av2 } 2}}\right)^{0.52}} \tag{12}
\end{equation*}
$$

for contacts with wavy surfaces.
For contacts made of metals with high thermal conductivity and surfaces having a waviness lower than class 3 and roughness lower than class 6 , the calculation of the TCR must take account of the fact that both macrophenomena and microphenomena take place in the zone of contact.

## NOTATION

$R^{\prime}$ is the total thermal resistance; $R$ is the specific thermal resistance; $R M$ is the thermal resistance of actual contact; $R_{c t r}$ is the thermal resistance of contraction; $\varphi$ is the coefficient of contraction; $\left.\lambda_{M}=\left(2 \lambda_{M_{1}} \lambda_{M_{2}}\right) / \lambda_{M_{1}}+\lambda_{M_{2}}\right)$ is the reduced thermal conductivity of the materials of the two contacting bodies; $n_{1}=S_{a} / S_{c}, n_{2}=S_{c} / S_{n}, n_{3}=$ $S_{a} / S_{n}$ are the relative contact areas; $S_{a}, S_{c}, S_{n}$ are the actual, contour, and nominal area of contact, respectively; $L$ is the wave interval; $d$ is the displacement of the
center of the contact area with respect to the axis of the cylindrical channel; $h$, $H_{m a}$ are the heights of roughness projection and wave, respectively; $P$ is the specific normal load; $\bar{E}=2 E_{1} E_{2} /\left(E_{1}+E_{2}\right)$ is the reduced modulus of normal elasticity; $\mu$ is the Poisson coefficient; $T_{c}$ is the temperature in the contact zone. Subscripts: 1 and 2 refer to first and second of the contacting bodies; ma, relative to macroscopic (contour) area of contact; mi, relative to microscopic area of contact; max, maximum; av, average.

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